

S Baer
Feb 2000

e and the Builder

Pocket Calculators have a key for e^x . Put in 1. $e^1 = e = 2.718\dots$ then find the logarithm of that same number by hitting the $\ln x$. Its log is 1. $\ln 2.718\dots = 1$.

Now you can forget, just the way you have forgotten the rest of calculus, but I hope you'll be interested to read more about e , the natural logarithm.

The fraction $1/e$, .37 is something we know vaguely in our bones, as we are familiar with $1/2$ and π , the ratio of circumference to diameter of a circle.

We know about the fraction $1/e$ because it measures disappointment, the share not done, at the deadline we ignorantly count on in so many different matters. Our schedule: 5, 4, 3, 2, 1, 0, (blast off); Nature's pace: 5, 4.09, 3.35, 2.74, 2.246, 1.84 (oops).

Lots of things don't do as they are told, they react to local conditions. Objects cool rapidly when they are hot, not so rapidly when they are merely warm. Progress on many tasks is the same. Certain subcontractors make progress, but never complete their task. Always they will "have it done in two weeks". Watch their progress and sure enough they work fast enough so that, were they not to slow, they would be done in two weeks. But they do slow (after all, they almost got it) just as the rate of cooling slows. The time to completion promised by the present rate of progress, whether to cool a pot of soup or clean a garage is called the time constant. After one time constant the job won't be done, $1/e$ will remain. It will never be done.

Illustration #1

Lots of mathematical relationships are only clear after you stop trying to count. It is a great shame for us that we learn to count so early and cling to counting so dearly.

Some progress goes in a straight line as if by momentum and can be counted. Enough is like this to keep us confused and disappointed with more complex matters which are not so simple.

Ventilating behaves the same as cooling. To blow all the smoke out of a space, it isn't enough to put in one fresh volume. The smoky air mixes with the fresh. After a fresh volume has entered, $1/e$ of the smoke remains, mixed with the fresh. It is still an "e"th (37%) as smoky as before, and after two volumes an "e"th of an "e"th (14%). Turn a corner with a trailer. The tongue is now at an angle. As you drive ahead the trailer straightens. Each time you drive one tongue length forward the trailer straightens to within $1/e$ (almost exactly). $1/e$ is everywhere, yet no patterns reveal themselves. The world is too complex, the roads aren't straight to straighten out on, and background temperatures steady to cool off to. The patterns show themselves only through study.

By chance I ran into the fraction $1/e$ while investigating means to shade direct sunlight while allowing daylight.

If I block the irritating direct sunlight then I also block most of the pleasant daylight. $1/e$, 37% of the daylight is the most such a shade allows through.

Fig. 2

Blocks all direct sun and all daylight.
Blocks all direct sun, allows $2 \times 2 = 1/4$ of daylight.

Blocks all direct sun. Allows $2/3 \times 2/3 \times 2/3 = .296$ of daylight.

Blocks all direct sun.

Allows $3/4 \times 3/4 \times 3/4 \times 3/4 = .31614$

Limit $N \rightarrow \infty \quad 64(N/N+1)^{N+1} = 1/e = .3679$

Other Matters

The equation $e^{\pi i} = -1$ is a profound puzzle - an amazement to dumbfound even the likes of Richard Feynman or Benjamin Pierce. It was a relief and amusement to read Conway's and Guy's The Book of Numbers where they explain that this is only a surprise we set up for ourselves, it should no more astonish us than finding that, after murdering our parents, one is an orphan.

What does mathematics mean for builders? The Spanish architect Gaudi made his forms using models hung upside down. The forms swung freely in tension catenary curves. The equation of a

catenary uses e , $Y = \frac{e^x - e^{-x}}{2}$. These, he inverted. Tension became compression. Gaudi's models

were their own computers. Thus, Gaudi went where mathematics invited and showed off

its beauty. In another spirit Gehry's museum in Bilbao has shapes unusual to us and to

mathematics. Fig. 3

Fig. 4

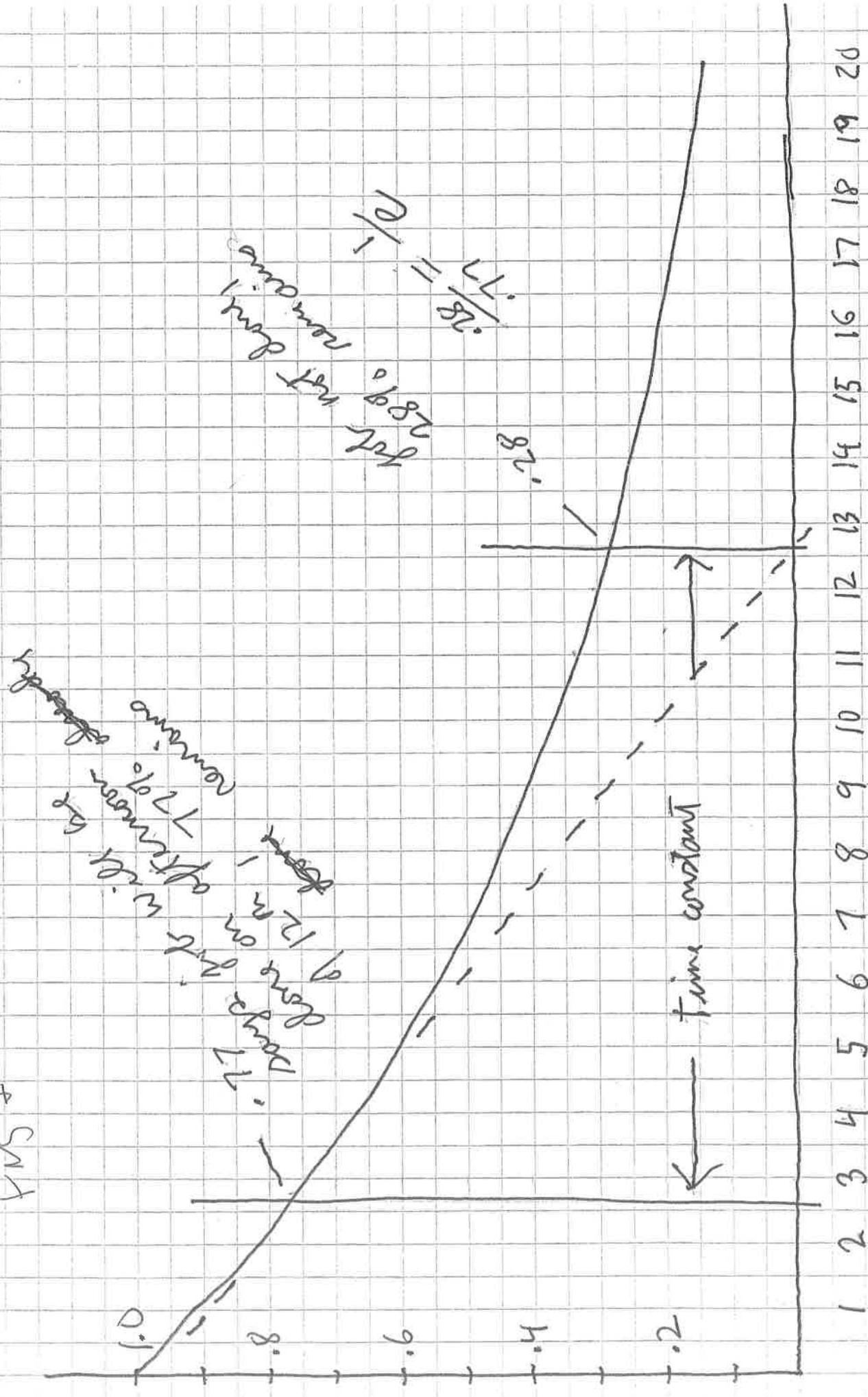
In Gaudi, form is mathematical perfection; in Gehry, a mathematical headache conquered by computer. Which appeals to you? Poor Gaudi, who went beyond counting with his suspended models may have neglected this sort of attention, for how else could a street car run over and kill

him? A street car also got L.J. Brouwer, the Dutch mathematician who challenged the law of the excluded middle.

The study of exponentials which explains the daylight, cooling, ventilation, and the path of the trailer clarifies only one tidbit of the world at a time and works best on things subsiding to equilibrium. Other things grow exponentially, as least for a time, but those who predict geometric explosions of either wealth or poverty usually deceive.

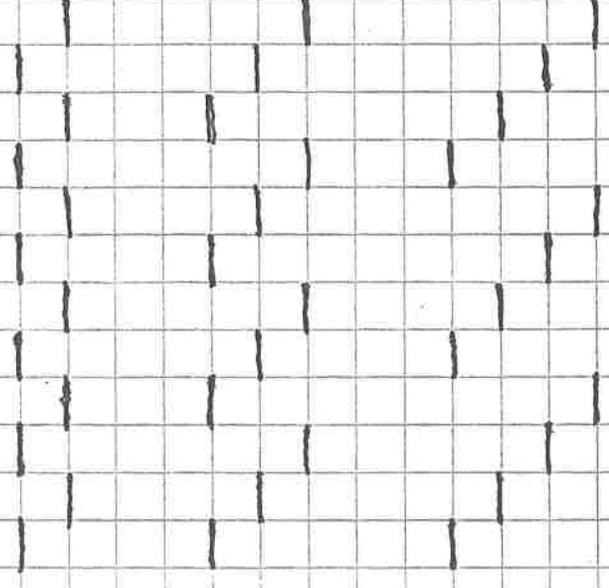
Sten Bar
Feb 2000

Figs 1



1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

Fish



$$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4} = .25$$

$$\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{8}{27} = .296$$

$$\frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} = \frac{81}{256} = .316$$

$$\left(\frac{N-1}{N}\right)^N \rightarrow \frac{1}{e} = .367...$$

who all own
directly owned
at it